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Neutral Particles in Light of the Majorana–Ahluwalia Ideas

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The first part of this article presents an overview of the theory and phenomenology of truly neutral particles based on the papers of Majorana, Racah, Furry, McLennan, and Case. The recent development of the construct undertaken by Ahluwalia could be relevant for the explanation of the present experimental situation in neutrino physics and astrophysics. Then the new fundamental wave equations for self-/anti-self-conjugate type II spinors proposed by Ahluwalia are recast into covariant form. The connection with Foldy–Nigam–Bargmann–Wightman–Wigner (FNBWW)-type quantum field theory is found. Possible applications to the problem of neutrino oscillations are discussed.

1. INTRODUCTION

Neutrino physics and astrophysics has put many dark spots in the cloudless sky of the Standard Model. For example, Robertson (1993) noted in this connection: "The solar neutrino results yield fairly strong and consistent indications that neutrino oscillations (Pontecorvo, 1957, 1958, 1967; Gribov and Pontecorvo, 1969; Bilen'kii and Pontecorvo, 1977, 1978; Bilen'kii, 1987; Bilen'kii and Petcov, 1987, 1989) are occurring."[3] Though "other evidence for new physics is less consistent and convincing," the solar neutrino problem (Langacker, 1994) [and in addition the "negative mass squared" problem (e.g., Robertson, 1993; Weinhammer *et al.*, 1993),[4] the atmospheric neutrino

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[3] There are opposite opinions on the solar neutrino problems. For example, Morrison (1994) denies their existence at all: "The evidence for any solar neutrino problem is not 'compelling'."

[4] See Lyubimov (1989) for an experiment in which the opposite result ($m_\nu^2 > 0$) was reached. See also Boris *et al.*, 1987.

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anomaly (Akmedov, 1994; Fukuda *et al.*, 1994), the possibility of neutrinoless double β-decay (Balysh *et al.*, 1992, 1994; Rosen, 1992), the "spin crisis" in QCD (Ashman *et al.*, 1988; Dorokhov *et al.*, 1993), the tentative experimental evidence for a tensor coupling in the $\pi^- \rightarrow e^- + \bar{\nu}_e + \gamma$ decay (Bolotov *et al.*, 1990),[5] as well as the dark matter problem (e.g., Binney and Tremaine, 1987; Griest, 1994)] provides sufficient reasons to search for models beyond the framework of the Standard Model. At the same time, the present experimental situation does not provide clear hints for theoreticians as to what principles should be used to explain the mentioned phenomena and to construct the "ultimate" theory. Thus, nature leaves us with many degrees of freedom in working out hypotheses which might seem at first sight to be "exotic"[6] if not "crazy" (Ahluwalia and Ernst, 1992; Dvoeglazov, 1994b–e).

In this paper I continue the study of $j = 1/2$ and $j = 1$ neutral particles (present knowledge state that the neutrino and the photon are the only truly neutral particles in nature) undertaken in Ahluwalia *et al.* (1994a,b) and Ahluwalia (1994a,b). The crucial point of those papers is "the dynamical role played by space-time symmetries for [fundamental] interactions". The

[5] For theoretical models see Chizhov (1993) and Chizhov and Avdeev (1994).

[6] A very exotic idea of nonzero electric charge of the neutrino has been proposed to explain the solar neutrino problem in Ignatiev and Joshi (1994a,b), based on Einstein's idea (see Piccard and Kessler (1925) of electric charge dequantization and its dependence on time (e.g., Babu and Mohapatra, 1989; Ignatiev and Joshi, 1993; Foot *et al.*, 1993a,b; Dvoeglazov, 1994e). For the possibility of the existence of mirror matter (e.g., a mirror photon with electric charge and/or mass) see Foot (1994a,b), Barr *et al.* (1991); see also Giveon and Witten (1994). According to Bandyopadhyay (1968a), "In view of the neutrino theory of light, photons are likely to interact weakly also, apart from the usual electromagnetic interactions. . . This assumed photon–neutrino weak interaction, if it exists, will have important bearing on astrophysics. In fact, this interaction can then be held responsible for the following neutrino-generating processes in stars:

(1) $\gamma + e^- \leftrightarrow e^- + \nu + \bar{\nu}$,

(2) $e^- + Z \leftrightarrow e^- + Z + \nu + \bar{\nu}$,

(3) $e^- + e^+ \leftrightarrow \nu + \bar{\nu}$,

(4) $\gamma + \gamma \leftrightarrow \nu + \bar{\nu}$,

(5) $\gamma + \gamma \leftrightarrow \gamma + \nu + \bar{\nu}$,

(6) $\Gamma \rightarrow \nu + \bar{\nu}$ ($\Gamma \rightarrow e^- + e^+ \rightarrow \gamma \rightarrow \nu + \bar{\nu}$) (*plasma process*).

. . .The energy dependence of the cross sections for these processes according to the present theory will be significantly different from that in other theories." Also see Bandyopadhyay (1968b). Ahluwalia *et al.* (1993) and Ahluwalia and Goldman are concerned with the theoretical construction of a Foldy–Nigam–Bargmann–Wightman–Wigner (FNBWW)-type quantum field theory; its remarkable features are that a boson can possess the opposite parity from its antiparticle, and a fermion and its antifermion can possess the same parities. Rembieliński (1994a,b) considers the possibility that neutrinos are fermionic tachyons (according to the present experimental data). Nevertheless, the principle of the "absolute causality" holds for all kind of events.

ab initio construction of self-/anti-self conjugate spinors in the $(j, 0) \oplus (0, j)$ representation space and derivation of some physically relevant properties connected with space-time symmetries were presented there. In fact, that work is the development of the formalism proposed in Majorana (1937), Racah (1937), Furry (1938, 1939), Serpe (1952), McLennan (1957), and Case (1957) and it could be applicable for the description of neutrino interactions and clarification of the present experimental situation.

2. THEORY AND PHENOMENOLOGY OF NEUTRAL PARTICLES

Kayser (1985) writes: "We have become accustomed to thinking of a neutrino ν and its antineutrino $\bar{\nu}$ as distinct particles.[7] However, it has long been recognized that the apparent distinction between them may be only an illusion. [Such] models [in which there is no difference between neutrino and its antineutrino] naturally follow from GUT (grand unification theories)." Moreover, from the viewpoint of many models beyond the Standard Model it is very natural for the neutrino (the spin-1/2 truly neutral particle) to be massive[8] (as opposed to the Glashow–Salam–Weinberg electroweak theory).

At this point I take the liberty of presenting a little history. Majorana (1937) gave a derivation of a symmetrical theory of the electron and the positron. The essential ingredient of that theory was the reformulation of the variational principle, based on the use of noncommutative variables. This led him to a separation of the Dirac equation "into two distinct groups, one of which acts on the real part and the other on the imaginary part of [the spinor wave function], $\Psi = U + iV$." He noted: "the part of this formalism which refers to the U (or to the V) may be considered by itself as a theoretical description of some material system, in conformity with the general methods of quantum mechanics. . . . Equations constitute the simplest theoretical representation of a system of neutral particles." His ideas were developed in application to β radioactivity by Racah (1937) and Furry (1938, 1939). In fact, they analyzed the Majorana projection[9]

$$\psi \rightarrow \frac{1}{2}\{\psi + S^c_{[1/2]}\psi\} \tag{1}$$

[7] Thanks to the two-component neutrino theory proposed by Landau (1957a,b), Lee and Yang (1957), and Salam (1957).

[8] Surprisingly, six of the present upper bounds on $m^2_{\nu_e}$ are negative (Gelmini and Roulet, 1994). For example, the LANL result is $-147 \pm 68 \pm 41$ eV^2 and the LLNL result is $-130 \pm 20 \pm 15$ eV^2. The most recent measurement (1994, Troitsk) involves a new kind of systematics and gives -18 ± 6 eV^2.

[9] The notation of Ahluwalia *et al.* (1994b) is used through the present paper, which is different from Racah (1937), Furry (1938, 1939), McLennan (1957), Case (1957), and Ryan and Okubo (1964).

The matrix of charge conjugation is defined as

$$S^c_{[1/2]} = e^{i\vartheta^c_{[1/2]}} \begin{pmatrix} 0 & i\Theta_{[1/2]} \\ -i\Theta_{[1/2]} & 0 \end{pmatrix} \mathcal{K} \equiv \mathcal{C}_{[1/2]}\mathcal{K} \tag{2}$$

where $\mathcal{K}$ is the operation of complex conjugation and

$$(\Theta_{[j]})_{\sigma,\sigma'} = (-1)^{j+\sigma}\,\delta_{\sigma',-\sigma} \tag{3}$$

is Wigner's operator ($\Theta_{[j]}\mathbf{J}\Theta^{-1}_{[j]} = -\mathbf{J}^*$). Racah noted that the symmetric description of a particle and an antiparticle does not always imply that two types of particle are physically undistinguishable. That is clear for the electron and the positron states, which have opposite electric charge, but this statement can also be applied for the neutrino: "a neutrino emitted in a β^- process may by absorption induce only a β^+ process, and vice versa." However, if we consider the symmetric Hamiltonian [the sum of H_F, the Fermi Hamiltonian, and H_{KU}, the Konopinski–Uhlenbeck Hamiltonian (Fermi, 1934; Konopinski and Uhlenbeck, 1935)], we come to the physical identity between neutrino and antineutrino and hence to the Majorana formalism for neutral particles, according to Racah, from which follows the experimental possibility of the neutrinoless double β decay discussed below. Furry (1938, 1939) proved the Lorentz invariance of the Majorana projection (1) as well as the persistence in time and the possibility of interaction of the Majorana particle with the nonelectric scalar potential $\gamma_0\Phi$. Furry also noted the noninvariance of the projection under the change of phase (i.e., in fact, with respect to multiplication by a complex constant, which implies the absence of the simple gauge interactions of the Majorana neutral particle as opposed to the Dirac charged particle). Differing from Racah, he has claimed that "the results predicted for ... observed processes [β-radioactivity] are ... identical with those of the ordinary theory. [However], the physical interpretation is quite different [and] an experimental decision between the formulation using neutrinos and antineutrinos and that using only neutrinos will ... be ... difficult."[10] His point of view is now widely accepted: as opposed to the Dirac prescription of the charged particle (which has four states which answer to the same momentum but different spin configurations of particle and antiparticle), in the Majorana theory for $j = 1/2$ particles there are just two states corresponding to

[10]Of course, in the case of massless states this assertion has not given rise to opposite opinions. Also, Ryan and Okubo (1964) claimed the equivalence of the descriptions of the neutrino in terms of Majorana spinors and Weil spinors, but their arguments implied zero neutrino mass. The very detailed pedagogical introduction of Mannheim (1984) to the Majorana theory includes a discussion of mass eigenstates of the neutrino. Nevertheless, in the case of massive neutrinos further explanation is required of the equivalence of the two descriptions and the question of the number of independent states. See also footnote 22 in Furry (1938); also Ahluwalia *et al.* (1994a,b), Ahluwalia (1994a,b), Sokolov (1957); and the discussion below.

the two projections of the spin, i.e., there are no "antiparticles" and no necessity of negative-energy states.

Important reformulations of Majorana's work have been undertaken by McLennan (1957) and Case (1957). Let me reproduce the main points of Case's paper.[11] By using the Majorana ansatz[12]

$$\psi_L = \mathscr{C}^{-1}_{[1/2]}\psi^*_R \tag{4}$$

where $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$, the Dirac equation can be rewritten as

$$\eta^\mu \partial_\mu \phi + \kappa\phi^* = 0 \tag{5}$$

and its complex conjugate

$$\eta^{\mu *}\partial_\mu \phi^* + \kappa\phi = 0 \tag{6}$$

Here $\eta^\mu = \mathscr{C}_{[1/2]}\gamma^\mu_- = \mathscr{C}_{[1/2]}(1 - \gamma_5)\gamma^\mu/2$, $\phi = \psi_R$, and κ is the mass of the particle in the notation of Case (1957). The matrices η^μ satisfy the anticommutation relation:

$$\eta^{\mu *}\eta^\nu + \eta^{\nu *}\eta^\mu = 2g^{\mu\nu} \tag{7}$$

The signature was chosen to be $(-1, +1, +1, +1)$. The corresponding Hamiltonian equations are

$$i\,\frac{\partial\phi}{\partial t} = \frac{1}{i}\,\boldsymbol{\sigma}\cdot\nabla\phi + \kappa(A\phi^*) \tag{8}$$

$$i\,\frac{\partial(A\phi^*)}{\partial t} = -\frac{1}{i}\,\boldsymbol{\sigma}\cdot\nabla(A\phi^*) + \kappa\phi \tag{9}$$

with $\eta^\mu = -iA\sigma^\mu$. The matrix A can be chosen as σ_2 in the conventional representation (Case, 1957, p. 308). The representation of the proper Lorentz transformation is, as usual, $\Lambda = \exp(\frac{1}{2}v\boldsymbol{\sigma}\cdot\mathbf{q})$ with velocity v in the direction $\mathbf{q}$. However, for spatial reflections one has to impose

$$\phi'(x') = \Lambda\phi^*(x), \qquad \text{or} \qquad \phi^*(x) = \Lambda^{-1}\phi'(x') \tag{10}$$

This form ensures that $\Lambda = i\rho A$, where ρ is a real number of absolute value unity. By using similar arguments for time reflections one has $\phi'(x') = \Lambda\phi^*(x)$, where $\Lambda = \mu A$, with μ being real (and its absolute value being equal to unity). However, the McLennan–Case consideration does not exhaust

[11] The papers of Serpe (1952) and McLennan (1957) are concerned with the massless neutrino and could be counted as particular cases. Let us not forget that we do not have a strong theoretical principle that forbids the mass of the neutrino.

[12] The definitions of K. M. Case and D. V. Ahluwalia differ by the overall phase factor.

all possible Majorana-like constructs. For instance, the possibility of the anti-self-conjugate construct, i.e.,

$$\psi \to \frac{1}{2}\{\psi - S^c_{[1/2]}\psi\} \tag{11}$$

was realized much later (Mannheim, 1984). From a physical point of view this corresponds to two neutrinos with opposite *CP* quantum numbers (e.g., Halprin *et al.*, 1976; Wolfenstein, 1981; Doi *et al.*, 1983).

Recently, the theory of neutral Majorana-like particles has been developed substantially by Ahluwalia (1994a,b; Ahluwalia *et al.*, 1994a,b). In particular, the generalization to higher spin particles has been proposed. The formalism is based on the type II bispinors (another Majorana-like construct which could be important for the description of higher spin particles) introduced by him. A fundamentally new wave equation was proposed there. We are going to discuss it in the next section.

The type II $(j, 0) \oplus (0, j)$ bispinors are defined in the following way:

$$\lambda(p^\mu) \equiv \begin{pmatrix} (\zeta_\lambda \Theta_{[j]})\phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix}, \qquad \rho(p^\mu) \equiv \begin{pmatrix} \phi_R(p^\mu) \\ (\zeta_\rho \Theta_{[j]})^* \phi_R^*(p^\mu) \end{pmatrix} \tag{12}$$

ζ_λ and ζ_ρ are phase factors that are fixed by the conditions of self-/anti-self conjugacy:

$$S^c_{[1/2]}\lambda(p^\mu) = \pm\lambda(p^\mu), \qquad S^c_{[1/2]}\rho(p^\mu) = \pm\rho(p^\mu) \tag{13}$$

for the $j = 1/2$ case, and

$$[\Gamma^5 S^c_{[1]}]\lambda(p^\mu) = \pm\lambda(p^\mu), \qquad [\Gamma^5 S^c_{[1]}]\rho(p^\mu) = \pm\rho(p^\mu) \tag{14}$$

for the $j = 1$ case.[13] The spin-1 counterpart of equation (2) is

$$S^c_{[1]} = e^{i\vartheta^c_{[1]}} \begin{pmatrix} 0 & \Theta_{[1]} \\ -\Theta_{[1]} & 0 \end{pmatrix} \mathcal{K} \equiv \mathcal{C}_{[1]}\mathcal{K} \tag{15}$$

The phase factors are determined as $\zeta^S_\lambda = \zeta^S_\rho = +i$ for the self-charge-conjugate $j = 1/2$ spinors $\lambda^S(p^\mu)$ and $\rho^S(p^\mu)$, and $\zeta^A_\lambda = \zeta^A_\rho = -i$ for the anti-self-charge-conjugate $j = 1/2$ spinors $\lambda^A(p^\mu)$ and $\rho^A(p^\mu)$. Equations (14) determine $\zeta^S_\lambda = \zeta^S_\rho = +1$ for the self $[\Gamma^5 S^c_{[1]}]$-conjugate $j = 1$ spinors, and $\zeta^A_\lambda = \zeta^A_\rho = -1$ for the anti-self $[\Gamma^5 S^c_{[1]}]$-conjugate $j = 1$ spinors. The remarkable property of the self-/anti-self-conjugate spinors, which seems not to have

[13] The self-/anti-self conjugate type II spinors were shown in Ahluwalia *et al.* (1994a,b) and Ahluwalia (1994a,b) not to exist for bosons. This fact is related to the FNBWW-type construction and it follows from the analysis of Ahluwalia *et al.* (1993). However, $[\Gamma^5 S^c]$ self-/anti-self conjugate type II spinors are introduced there.

been realized before the work of Ahluwalia, is that they cannot be in definite helicity eigenstates. In fact, let the 2-spinors $\phi^h_{L,R}(p^\mu)$ be an eigenstate of the helicity operator

$$\mathbf{J}\cdot\hat{\mathbf{p}}\ \phi^h_{L,R}(p^\mu) = h\phi^h_{L,R}(p^\mu) \tag{16}$$

Then, by using the Wigner identity [see equation (3)], we can convince ourselves that

$$\mathbf{J}\cdot\hat{\mathbf{p}}\ \Theta_{[j]}[\phi^h_{L,R}(p^\mu)]^* = -h\Theta_{[j]}[\phi^h_{L,R}(p^\mu)]^* \tag{17}$$

Thus, if $\phi^h_{L,R}(p^\mu)$ are eigenvectors of $\mathbf{J}\cdot\hat{\mathbf{p}}$, then $\Theta_{[j]}[\phi^h_{L,R}(p^\mu)]^*$ are eigenvectors of $\mathbf{J}\cdot\hat{\mathbf{p}}$ with *opposite* eigenvalues to those associated with $\phi^h_{L,R}(p^\mu)$ (Ahluwalia, 1994a,b). The unusual properties of the type II spinors under space (time) reflections have also been noted by Ahluwalia and co-workers. They are not eigenspinors of the parity operator; see formulas (36a,b) and (37a,b) in Ahluwalia (1994b).

The key test for a Majorana neutrino is neutrinoless double-beta decay. An antineutrino emitted in the beta decay of one neutron is supposed to interact with another neutron and to cause it to transform into a proton and an electron. So in the final state there are two protons, two electrons, and no neutrinos, $(A, Z) \to (A, Z + 2) + 2e^-$. The conservation of lepton number is violated. Such a possibility, originally proposed by Racah (1937), has not yet been observed in experiment in spite of the fact that the available phase space for this process is larger than for the two-neutrino double-β decay (Furry, 1938, 1939). The experimental bound for the half-life of neutrinoless β decay is $T_{1/2} > 2 \times 10^{24}$ years [the enriched isotope ^{76}Ge was used (Balysh, 1992, 1994)]. The failure to observe it was explained by stating that apart from the nonconservation of lepton number, the Racah processes is inhibited by helicity. In order to complete the second step of the Racah process, the antineutrino has to flip its helicity and turns itself into a neutrino.[14] Rosen (1992) has shown that such a flip may be induced only by a Majorana mass term: "Even if right-handed currents provide the phenomenological mechanism for no-neutrino decay, the fundamental mechanism underlying the process must be [the presence of] neutrino mass [term]." In the case of neutral particles electric charge conservation (superselection rules) no longer forbids transitions between particle and antiparticle $\nu_{eL} \leftrightarrow \bar{\nu}_{eR}$ or $\bar{\nu}_{eL} \leftrightarrow \nu_{eR}$. It is these oscillations that provide the ground for the Racah process. The first theoretical model of neutrino oscillations was proposed by Pontecorvo (1957)[15] (see also Maki *et al.*, 1962), by using the analogy with oscillations

[14]Of course, this explanation is appropriate only in the framework of the Standard Model.
[15]As mentioned in Rosen (1992), some rumors of a positive result concerning no-neutrino decay were circulated at the end of the 1950s.

in the K^0–$\bar{K}^0$ spinless meson system (Gell-Mann and Pais, 1955; Pais and Piccioni, 1955). This old idea eventually went out of use, but it has found new life in the idea of oscillations between different flavors (Gribov and Pontecorvo, 1969; Bilen'kii and Pontecorvo, 1977, 1978; Bilen'kii, 1987; Bilen'kii and Petcov, 1987, 1989; Pontecorvo, 1971; Bilenky and Pontecorvo, 1976a,b; Fritzsch and Minkowski, 1976; Eliezer and Swift, 1976) in connection with the discovery of muon and τ-lepton neutrinos.

Since in the Section 3 I am going to deal with a scheme of neutrino oscillations on the basis of a Majorana-like theory with type II spinors, let me reproduce here the main points of the well-known flavor mixing scheme[16] and of the commonly used consideration of neutrino mass terms.

Schemes of neutrino mixing are usually characterized by the type of the relevant mass term. According to the modern literature it is possible to form the following mass terms in the Lagrangian[17]:

- Dirac mass term:

$$\mathcal{L}^{\mathrm{D}} = - \sum_{l',l=e,\mu,\tau\ldots} \bar{\nu}_{l'R} M_{l'l} \nu_{lL} + \mathrm{h.c.} \tag{18}$$

- Majorana mass term (left–left):

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} \sum_{l',l=e,\mu\tau\ldots} \overline{(\nu_{l'L})^c} M_{l'l} \nu_{lL} + \mathrm{h.c.} \tag{19}$$

- Dirac plus Majorana mass term:

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \sum_{l',l=e,\mu,\tau\ldots} \overline{(\nu_{l'L})^c} M^L_{l'l} \nu_{lL} - \sum_{l',l=e,\mu,\tau\ldots} \bar{\nu}_{l'R} M^{\mathrm{D}}_{l'l} \nu_{lL}$$
$$- \frac{1}{2} \sum_{l',l=e,\mu,\tau\ldots} \overline{\nu_{l'R}} M^R_{l'l} (\nu_{lR})^c + \mathrm{h.c.} \tag{20}$$

So, in the general case it is necessary to consider three (six) mass eigenstates that correspond to the diagonalized mass matrix obtained by the unitary transformation with the $3 \otimes 3$ (or $6 \otimes 6$ in the case of the D + M mass term) matrix, e.g., $\nu_{lL} = \sum_{i=1}^{3} U_{li} \nu_{iL}$. We will denote the mass eigenstates

[16]More extended consideration can be found in Bilen'kii and Ponteciorvo (1977, 1978), Bilen'kii (1987), Bilen'kii and Petcov (1987, 1989), Hughes (1991), and Rolnick (1994).

[17]The present experimental data restrict the number of light neutrino species to three (electron, muon, and τ-lepton neutrino) (Adriani *et al.*, 1992). The astrophysical limit $N_\nu \leq 4$ was given in Steigman *et al.* (1986).

$|\nu_i\rangle$, $i = 1, 2, 3$. Thus, one can obtain the diagonalized mass term in the Lagrangian:

$$\mathcal{L}^{\mathrm{D}} = -\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i \tag{21}$$

$$\mathcal{L}^{\mathrm{M(D+M)}} = -\frac{1}{2} \sum_{i=1}^{3(6)} m_i \bar{\chi}_i \chi_i \tag{22}$$

The most general mass matrix (Dirac and Majorana mass term) can be represented in the following form:

$$\overline{\Psi_{L,R}} M \Psi_{L,R} = (\bar{\psi}_L \quad (\bar{\psi}_R)^c \quad (\bar{\psi}_L)^c \quad \bar{\psi}_R) \begin{pmatrix} 0 & 0 & m_L & m_D \\ 0 & 0 & m_D & m_R \\ m_L & m_D & 0 & 0 \\ m_D & m_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ (\psi_R)^c \\ (\psi_L)^c \\ \psi_R \end{pmatrix} \tag{23}$$

In the vacuum, mass eigenstates propagate independently, i.e., let assume that they are orthogonal. If a physical state is the linear combination of mass eigenstates which have different masses (for the sake of simplicity we consider only two species), one has

$$\begin{cases} |\nu_e(0)\rangle = \cos\theta_\nu\,|\nu_1\rangle + \sin\theta_\nu\,|\nu_2\rangle \\ |\nu_\mu(0)\rangle = -\sin\theta_\nu\,|\nu_1\rangle + \cos\theta_\nu\,|\nu_2\rangle \end{cases} \tag{24}$$

The partial content of species in it may vary with time. Suppose that at time $t = 0$ we have the mixing (24); then, at a later time t,

$$\begin{aligned} |\nu_e(t)\rangle &= \cos\theta_\nu\, e^{-iE_1t}|\nu_1\rangle + \sin\theta_\nu\, e^{-iE_2t}|\nu_2\rangle \\ &= (e^{-iE_1t}\cos^2\theta_\nu + e^{-E_2t}\sin^2\theta_\nu)|\nu_e(0)\rangle \\ &\quad + \sin\theta_\nu\cos\theta_\nu\,(e^{-iE_2t} - e^{-E_1t})|\nu_\mu(0)\rangle \end{aligned} \tag{25}$$

and

$$\begin{aligned} |\nu_\mu(t)\rangle &= -\sin\theta_\nu\, e^{-iE_1t}|\nu_1\rangle + \cos\theta_\nu\, e^{-E_2t}|\nu_2\rangle \\ &= \sin\theta_\nu\cos\theta_\nu\,(e^{-E_2t} - e^{-iE_1t})|\nu_e(0)\rangle \\ &\quad + (e^{-iE_2t}\cos^2\theta_\nu + e^{-E_1t}\sin^2\theta_\nu)|\nu_\mu(0)\rangle \end{aligned} \tag{26}$$

Thus, an electron neutrino produced at $t = 0$ has nonzero probability of being a muon neutrino at a later time (and vice versa). The probability is calculated to give

$$\begin{aligned} P_{\nu_e \to \nu_\mu} &= |\langle \nu_\mu(0) | \nu_e(t) \rangle|^2 \\ &= |\sin\theta_\nu \cos\theta_\nu \, (e^{-iE_2 t} - e^{-iE_1 t})|^2 \\ &= 2\sin^2\theta_\nu \cos^2\theta_\nu \, [1 - \cos(E_1 - E_2)t] \end{aligned} \tag{27}$$

For the sake of completeness let us note that

$$P_{\nu_e \to \nu_e} = |\langle \nu_e(0) | \nu_e(t) \rangle|^2 = 1 - \sin^2 2\theta_\nu \sin^2\left[\frac{1}{2}(E_2 - E_1)t\right] \tag{28}$$

Since in the high-velocity limit ($p >> m$)

$$E_1 - E_2 = (p^2 + m_1^2)^{1/2} - (p^2 + m_2^2)^{1/2} \approx \frac{m_1^2 - m_2^2}{2p} \tag{29}$$

one obtains

$$P_{\nu_e \to \nu_\mu} \approx 2\sin^2\theta_\nu \cos^2\theta_\nu \left[1 - \cos\left(\frac{m_1^2 - m_2^2}{2p}\right)\frac{c^3}{\hbar}\, t\right] \tag{30}$$

where we restored c and $\hbar$ in order for the cosine to be dimensionless. Since the velocity of the neutrino is approximately (?) equal to the light velocity, one has

$$\begin{aligned} P_{\nu_e \to \nu_\mu} &\approx 2\sin^2\theta_\nu \cos^2\theta_\nu \left[1 - \cos\left(\frac{m_1^2 - m_2^2}{2p}\right)\frac{c^2}{\hbar}\, x\right] \\ &= 2\sin^2\theta_\nu \cos^2\theta_\nu \left(1 - \cos 2\pi \frac{x}{l_{12}}\right) \end{aligned} \tag{31}$$

where

$$l_{12} = \frac{4\pi p \hbar}{(m_1^2 - m_2^2)c^2} \tag{32}$$

is the "vacuum oscillation length." In the case of almost "degenerate" neutrinos $(m_1^2 - m_2^2) \approx (10^{-2}\ \mathrm{eV}/c^2)^2$ the "oscillation length" l_{12} is of the order of meters. For the numerous literature on other versions of the oscillations (including three species, etc.), see a recent review (Gelmini and Roulet, 1994).

Present-day experiments have not detected any such oscillations for terrestrially (nuclear reactors, accelerators) created neutrinos. This is usually explained by the very small mass differences between eigenstates. On the other hand, the study of solar neutrinos reveals a strong possibility that, before they reach the earth, the neutrinos undergo a significant oscillation. Besides vacuum oscillations, plasma processes also should be taken into account in the analysis of the solar neutrino flux. However, we are not going to discuss here the transmission through matter [the Mikheyev–Smirnov–Wolfenstein effect (Wolfenstein, 1978; Mikheyev and Smirnov, 1986a,b,c)], referring the reader to the review by Kuo and Panteleone (1989).

For the moment, many physicists do not consider seriously Pontecorvo's original idea. "Since the helicity of a free particle is conserved, in vacuum the oscillations $\nu_L \rightarrow \bar{\nu}_R$ cannot occur. . . For the above reason it was generally supposed that Pontecorvo's original oscillations are just the oscillations of active neutrinos into sterile states [e.g., $\nu_L \rightarrow \bar{\nu}_L$], whereas the true neutrino–antineutrino oscillations were considered impossible" (Akhmedov *et al.*, 1993).[18] Nevertheless, the same authors realized that under certain conditions particle–antiparticle oscillations can occur and revisited the original idea on the basis of the introduction of magnetic (or electric) dipole moment of the neutrino with the addition of a neutrino of the other species. A similar conclusion was reached by Hughes (1991, p. 378), who said that "traversal of the solar magnetic field may flip the neutrino spin." However, the estimated order of the transition magnetic moment is $\mu_\nu \sim 10^{-11}$–$10^{-10}\ \mu_B$. "[Nevertheless], resonant effects in a full treatment may well enhance the spin-flip to a level where it is important."

The history of the Majorana theory (as well as of neutrino physics itself) is very dramatic: one can see from the above that many outstanding physicists were not able to find common answers on the experimental consequences of this description.

Next, in the following section we shall work with spin-1 fields in the Weinberg formulation. Therefore, it is useful to repeat the key points of this *particular* model presented in (Weinberg, 1964a,b, 1969; Sankaranarayanan and Good, 1965a,b; Sankaranarayanan, 1965; Tucker and Hammer, 1971; Ahluwalia *et al.*, 1993; Ahluwalia and Goldman, 1993; Dvoeglazov, 1993b, 1994a–e). The pioneering study of the $(j, 0) \oplus (0, j)$ representation space for description of higher spin particles was undertaken by Weinberg (1964a,b, 1969). This approach is on an equal footing with Dirac's description of spin-1/2 particles and, in fact, has its origin from Wigner's (1939) classic work. In the Weinberg theory a $2(2j + 1)$ bispinor is constructed from left- and

[18]Compare Akhmedov *et al.* (1993) and Rosen (1992, pp. 4, 5) on neutrino–antineutrino oscillations.

right-spinors ϕ_L and ϕ_R transforming according to the $(j, 0) \oplus (0, j)$ representation of the Lorentz group. Without reference to any wave equation, it can be shown that

$$(j, 0): \quad \phi_R(p^\mu) = \Lambda_R(p^\mu \leftarrow \mathring{p}^\mu)\phi_R(\mathring{p}^\mu) = \exp(+\mathbf{J}\cdot\boldsymbol{\varphi})\ \phi_R(\mathring{p}^\mu) \tag{33}$$

$$(0, j): \quad \phi_L(p^\mu) = \Lambda_L(p^\mu \leftarrow \mathring{p}^\mu)\phi_L(\mathring{p}^\mu) = \exp(-\mathbf{J}\cdot\boldsymbol{\varphi})\ \phi_L(\mathring{p}^\mu) \tag{34}$$

where Λ_L and Λ_R are Lorentz boost matrices for left and right j-dimensional spinors from the rest system $\mathring{p}^\mu$; $\boldsymbol{\varphi}$ are the Lorentz boost parameters; the operator $\mathbf{J}$ is given by the angular momentum matrices. The Weinberg equation contains solutions with tachyonic dispersion relations.[19] Tucker and Hammer (1971) showed that it is possible to reformulate the $2(2j + 1)$ theory and to obtain spin-j equations which possess the *correct* physical dispersion. Positive- and negative-energy spinors coincide in their construct. However, the introduction of electromagnetic gauge interaction in their equation for $j = 1$ mesons appears to be difficult. The resulting theory is not renormalizable for $j \geq 1$. Another reformulation has been recently proposed. Based on the analysis of the transformation properties of left and right spinors and a choice of appropriate rest spinors (spinorial basis), Ahluwalia *et al.* (1993) noted that it is possible to construct a *Dirac*-like theory in $(j, 0) \oplus (0, j)$ space for arbitrary spin j (also see Sankaranarayanan and Good, 1965a,b). The remarkable feature of this construct is that the boson and its antiboson have opposite relative intrinsic parities. Such a theory has been named the Foldy–Nigam–Bargmann–Wightman–Wigner (FNBWW)-type quantum field theory. Finally, in Dvoeglazov (1994b–e) I give another Weinberg–Tucker–Hammer equation ("Weinberg double") with a correct physical dispersion. These equations turn out to be equivalent to the equations for the antisymmetric tensor $F_{\mu\nu}$ and its dual, which can be deduced from the Proca theory. The field consideration of the Weinberg doubles partly clarifies contradictions with

[19]The massless *first-order* "Weinberg" equations for any spin are proven in Ahluwalia and Ernst (1992), Table 2, to possess other kinematical acausalities. In addition to the correct physical dispersion $E = \pm p$, there is a wrong dispersion relation $E = 0$ in the case $j = 1$ (in the case of higher spins one has even more acausal solutions). This cast doubt on their application for all processes (including quantum electrodynamic processes). Nevertheless, the massless limits of the modified $2j$-order Weinberg equations ($\wp_{u,v} = \pm 1$ for bosons)

$$[\gamma^{\mu_1\mu_2\cdots\mu_{2j}}\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_{2j}} + \wp_{u,v} m^{2j}]\ \Psi(x) = 0 \tag{35}$$

turn out to be well defined and have no kinematical acausality (Ahluwalia and Ernst, 1992). The γ-matrices are covariantly defined $2(2j + 1) \otimes 2(2j + 1)$ matrices. See also Dvoeglazov (1993a,b, 1994a–e) and Dvoeglazov and Khudyakov (1994) for discussion of the connection of the Weinberg formulation with the antisymmetric tensor field description and for attempts to explain the origins and the consequences of incorrect dispersion relations.

the Weinberg theorem[20] in Hayashi (1973), Kalb and Ramond (1974), and Dvoeglazov (1993, 1994a). The contradictions were caused by the application of the generalized Lorentz condition [formulas (18) of Hayashi (1973)] to physical quantum states, which resulted in equating the eigenvalues of the Pauli–Lyuban'sky operator to zero. The propagators for the Weinberg–Tucker–Hammer construct have also been obtained (Dvoeglazov, 1994d).

However, these new constructs deal with Dirac-type spinors (type I spinors) and they are applicable mainly to charged particles. Many questions related to neutral particles are left unsolved in Ahluwalia *et al.* (1993), Ahluwalia and Goldman (1993), and Dvoeglazov (1994b–e).

3. NEW FUNDAMENTAL EQUATION PROPOSED BY AHLUWALIA AND RELEVANT PHYSICAL CONSEQUENCES

The general wave equation for any spin in the instant-front formulation of QFT is given in Ahluwalia (1994a,b) as[21]

$$\begin{pmatrix} -\mathbb{1} & \zeta_\lambda \exp(\mathbf{J}\cdot\boldsymbol{\varphi})\, \Theta_{[j]}\Xi_{[j]} \exp(\mathbf{J}\cdot\boldsymbol{\varphi}) \\ \zeta_\lambda \exp(-\mathbf{J}\cdot\boldsymbol{\varphi})\, \Xi_{[j]}^{-1}\Theta_{[j]} \exp(-\mathbf{J}\cdot\boldsymbol{\varphi}) & -\mathbb{1} \end{pmatrix} \lambda(p^\mu) = 0 \quad (36)$$

The particular cases ($j = 1/2$ and $j = 1$) are also given there [equations (31) and (32), respectively]. The $\lambda^S(p^\mu)$ appear to be the positive-energy solutions with $E = +(m^2 + \mathbf{p}^2)^{1/2}$, the $\lambda^A(p^\mu)$, negative-energy solutions with $E = -(m^2 + \mathbf{p}^2)^{1/2}$ for both spin-1/2 and spin-1 cases. However, to rewrite these equations in a covariant form is a difficult task. For instance, an attempt by Ahluwalia to put the equation in the form $(\lambda^{\mu\nu}p_\mu p_\nu + m\lambda^\mu p_\mu - 2m^2\mathbb{1})\lambda(p^\mu) = 0$ was in a certain sense misleading. He noted, "it turns out that [matrices] $\lambda^{\mu\nu}$ and λ^μ do not transform as Poincaré tensors." Below I try to explain how the equations for $\lambda(p^\mu)$ and $\rho(p^\mu)$ spinors can be rewritten in a covariant form.

The crucial point in the derivation of equation (36) is the generalized Ryder–Burgard relation for type II spinors,[22]

$$[\phi_L^h(\mathring{p}^\mu)]^* = \Xi_{[j]}\phi_L^h(\mathring{p}^\mu) \quad (37)$$

[20] The Weinberg theorem says that for massless particles $B - A = helicity$ if the field transforms on the (A, B) Lorentz group representation.

[21] See the corresponding equation in the light-front formulation in Ahluwalia *et al.* (1994a).

[22] Ahluwalia *et al.* (1993) and Ahluwalia and Goldman (1993) call the relation $\phi_R(\mathring{p}^\mu) = \pm\phi_L(\mathring{p}^\mu)$ for type I spinors (in fact, for the Dirac bispinor) the Ryder–Burgard relation; see also Ryder (1987, p. 44). Throughout this paper I also use this name, but this relation can be found in earlier papers and books; see e.g., the discussion surrounding equations (25, 26) of Chapter 5 of Novozhilov (1975). It can be deduced also from equation (22a) of Faustov (1971).

where

$$\Xi_{[1/2]} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}, \qquad \Xi_{[1]} = \begin{pmatrix} e^{i2\phi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i2\phi} \end{pmatrix} \tag{38}$$

h is the helicity, and ϕ is the azimuthal angle associated with $\mathbf{p}$. In this framework ($j = 1/2$ case) the best that can be done is to rewrite (36) in the form

$$\left(\frac{i\zeta_\lambda}{\sin\theta}\gamma_5[\gamma \times \hat{\mathbf{p}}]_3 + \mathbb{1}\right)\lambda(p^\mu) = 0 \tag{39}$$

(θ is the polar angle associated with $\mathbf{p}$) by using the identities

$$\Theta_{[1/2]}\Xi_{[1/2]} = \Xi^{-1}_{[1/2]}\Theta_{[1/2]} = i\sigma_1 \sin\phi - i\sigma_2 \cos\phi = i\frac{[\boldsymbol{\sigma} \times \mathbf{p}]_3}{(p_1^2 + p_2^2)^{1/2}} \tag{40}$$

$$[\boldsymbol{\sigma} \times \mathbf{p}](\boldsymbol{\sigma}\cdot\mathbf{p}) = -(\boldsymbol{\sigma}\cdot\mathbf{p})[\boldsymbol{\sigma} \times \mathbf{p}] = i\boldsymbol{\sigma}\mathbf{p}^2 - i\mathbf{p}(\boldsymbol{\sigma}\cdot\mathbf{p}) \tag{41}$$

and

$$\exp(\pm\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}/2) = \cosh(\varphi/2) \pm (\boldsymbol{\sigma}\hat{\boldsymbol{\varphi}})\sinh(\varphi/2), \qquad \hat{\boldsymbol{\varphi}} = \hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}| \tag{42}$$

However, the obtained equation cannot be considered as a dynamical equation (the energy operator is not present). In fact, (39) is only a reformulation of the condition of self-/anti-self-conjugacy.

Let us undertake another attempt. From the analysis of the rest spinors [formulas (22a)–(22b) and (23a)–(23c) of Ahluwalia (1994b)] one can conclude that another form of the generalized Ryder–Burgard relation is possible. Namely, the form connecting 2-spinors of the opposite helicity is

$$[\phi_L^h(\mathring{p}^\mu)]^* = (-1)^{1/2-h}e^{-i(\theta_1+\theta_2)}\Theta_{[1/2]}\phi_L^{-h}(\mathring{p}^\mu) \tag{43}$$

for the $j = 1/2$ case, and

$$[\phi_L^h(\mathring{p}^\mu)]^* = (-1)^{1-h}e^{-i\delta}\Theta_{[1]}\phi_L^{-h}(\mathring{p}^\mu) \tag{44}$$

for the $j = 1$ case ($\delta = \delta_1 + \delta_3$ for $h = \pm 1$ and $\delta = 2\delta_2$ for $h = 0$). Provided that the overall phase factors of the rest spinors are chosen to be $\theta_1 + \theta_2 = 0$ (or 2π) in the spin-1/2 case and $\delta_1 + \delta_3 = 0 = \delta_2$ in the spin-1 case, the Ryder–Burgard relation is written

$$[\phi_L^h(\mathring{p}^\mu)]^* = (-1)^{j-h}\Theta_{[j]}\phi_L^{-h}(\mathring{p}^\mu) \tag{45}$$

This choice is convenient for calculations. The same relations exist for right-handed spinors $\phi_R(\mathring{p}^\mu)$ in both the $j = 1/2$ case and the $j = 1$ case.

By using (45) and following the procedure of deriving the wave equation developed in Ahluwalia *et al.* (1994a,b) and Ahluwalia (1994a,b), one can obtain for the $j = 1/2$ case ($\hat{p} = \gamma^\mu p_\mu$)

$$\left[\frac{i}{m}\gamma_5\hat{p} - 1\right]\Psi^{(S)}_{+1/2}(p^\mu) = 0, \qquad \left[\frac{i}{m}\gamma_5\hat{p} + 1\right]\Psi^{(A)}_{+1/2}(p^\mu) = 0 \tag{46}$$

$$\left[\frac{i}{m}\gamma_5\hat{p} + 1\right]\Psi^{(S)}_{-1/2}(p^\mu) = 0, \qquad \left[\frac{i}{m}\gamma_5\hat{p} - 1\right]\Psi^{(A)}_{-1/2}(p^\mu) = 0 \tag{47}$$

Here we defined new spinor functions:

$$\Psi^{(S)}_{+1/2}(p^\mu) = \begin{pmatrix} i\Theta_{1/2}[\phi_L^{-1/2}(p^\mu)]^* \\ \phi_L^{+1/2}(p^\mu) \end{pmatrix} \quad \text{or} \quad \Psi^{(S)}_{+1/2}(p^\mu) = -i\begin{pmatrix} \phi_R^{+1/2}(p^\mu) \\ -i\Theta_{1/2}[\phi_R^{-1/2}(p^\mu)]^* \end{pmatrix} \tag{48}$$

$$\Psi^{(S)}_{-1/2}(p^\mu) = \begin{pmatrix} i\Theta_{1/2}[\phi_L^{+1/2}(p^\mu)]^* \\ \phi_L^{-1/2}(p^\mu) \end{pmatrix} \quad \text{or} \quad \Psi^{(S)}_{-1/2}(p^\mu) = i\begin{pmatrix} \phi_R^{-1/2}(p^\mu) \\ -i\Theta_{1/2}[\phi_R^{+1/2}(p^\mu)]^* \end{pmatrix} \tag{49}$$

$$\Psi^{(A)}_{+1/2}(p^\mu) = \begin{pmatrix} -i\Theta_{1/2}[\phi_L^{-1/2}(p^\mu)]^* \\ \phi_L^{+1/2}(p^\mu) \end{pmatrix} \quad \text{or} \quad \Psi^{(A)}_{+1/2}(p^\mu) = i\begin{pmatrix} \phi_R^{+1/2}(p^\mu) \\ i\Theta_{1/2}[\phi_R^{-1/2}(p^\mu)]^* \end{pmatrix} \tag{50}$$

$$\Psi^{(A)}_{-1/2}(p^\mu) = \begin{pmatrix} -i\Theta_{1/2}[\phi_L^{+1/2}(p^\mu)]^* \\ \phi_L^{-1/2}(p^\mu) \end{pmatrix} \quad \text{or} \quad \Psi^{(A)}_{-1/2}(p^\mu) = -i\begin{pmatrix} \phi_R^{-1/2}(p^\mu) \\ i\Theta_{1/2}[\phi_R^{+1/2}(p^\mu)]^* \end{pmatrix} \tag{51}$$

As opposed to $\lambda(p^\mu)$ and $\rho(p^\mu)$, these spinor functions are the eigenfunctions of the helicity operator of the $(1/2, 0) \oplus (0, 1/2)$ representation space, but they are not self-/anti-self-conjugate spinors.

Equations similar to (46, 47) can also be obtained by the procedure described in footnote 1 of Ahluwalia (1994b) with type I spinors (Ψ = column($\phi_R(p^\mu)$ $\phi_L(p^\mu)$)) if we take the Ryder–Burgard relation in the form

$$\phi_R(\mathring{p}^\mu) = \pm i\phi_L(\mathring{p}^\mu) \tag{52}$$

Equations of the kind (46, 47) have been discussed in the literature (Sokolik, 1957). Their relevance to the problem of describing the neutrino has been noted in the cited paper. The properties of these bispinors with respect to the parity (γ_0) operation are the following [cf. formulas (36a,b) in Ahluwalia (1994b)]:

$$\gamma_0\Psi^{(S)}_{+1/2}(p'^\mu) = -i\{\Psi^{(A)}_{-1/2}(p^\mu)\}^c \tag{53}$$

$$\gamma_0\Psi^{(S)}_{-1/2}(p'^\mu) = +i\{\Psi^{(A)}_{+1/2}(p^\mu)\}^c \tag{54}$$

$$\gamma_0 \Psi^{(A)}_{+1/2}(p'^\mu) = -i\{\Psi^{(S)}_{-1/2}(p^\mu)\}^c \tag{55}$$

$$\gamma_0 \Psi^{(A)}_{-1/2}(p'^\mu) = +i\{\Psi^{(S)}_{+1/2}(p^\mu)\}^c \tag{56}$$

By using the formulas relating Ψ, equations (48)–(51), with self-/anti-self-conjugate spinors, it is easy to find corresponding equations for spinors $\lambda(p^\mu)$ and $\rho(p^\mu)$. In the case of the spin-1/2 field we obtain

$$\hat{p}\lambda^S_\uparrow(p^\mu) + im\lambda^S_\downarrow(p^\mu) = 0, \qquad \hat{p}\rho^S_\uparrow(p^\mu) - im\rho^S_\downarrow(p^\mu) = 0 \tag{57}$$

$$\hat{p}\lambda^S_\downarrow(p^\mu) - im\lambda^S_\uparrow(p^\mu) = 0, \qquad \hat{p}\rho^S_\downarrow(p^\mu) + im\rho^S_\uparrow(p^\mu) = 0 \tag{58}$$

$$\hat{p}\lambda^A_\uparrow(p^\mu) - im\lambda^A_\downarrow(p^\mu) = 0, \qquad \hat{p}\rho^A_\uparrow(p^\mu) + im\rho^A_\downarrow(p^\mu) = 0 \tag{59}$$

$$\hat{p}\lambda^A_\downarrow(p^\mu) + im\lambda^A_\uparrow(p^\mu) = 0, \qquad \hat{p}\rho^A_\downarrow(p^\mu) - im\rho^A_\uparrow(p^\mu) = 0 \tag{60}$$

(provided that $m \neq 0$). The indices $\uparrow$ or $\downarrow$ should be referred to the chiral helicity introduced in Ahluwalia (1994a, p. 10). If we take similarly to Ahluwalia (1994b) $\lambda^S_{\uparrow\downarrow}(p^\mu)$, [and $\rho^A_{\uparrow\downarrow}(p^\mu)$] as the positive-energy solutions and $\lambda^A_{\uparrow\downarrow}(p^\mu)$ [and $\rho^S_{\uparrow\downarrow}(p^\mu)$] as the negative-energy solutions, we can write (57)–(60) in the coordinate space

$$\partial_\mu \gamma^\mu \lambda_\eta(x) + \not{\wp}_{\uparrow\downarrow} m \lambda_{-\eta}(x) = 0 \tag{61}$$

$$\partial_\mu \gamma^\mu \rho_\eta(x) + \not{\wp}_{\uparrow\downarrow} m \rho_{-\eta}(x) = 0 \tag{62}$$

where $\not{\wp}_{\uparrow\downarrow} = \pm 1$, with the plus sign if $\eta = \uparrow$ and the minus sign if $\eta = \downarrow$. These equations (61) and (62) are very similar to the Dirac equation; however, the sign of the mass term can be opposite and spinors enter in the equations with opposite chiral helicities. The Dirac equation with the opposite sign for the mass term has been considered (in different aspects) in (Markov, 1937, 1963, 1964; Belinfante, 1939; Belinfante and Pauli, 1940; Brana and Ljolje, 1980). Equations (61), (62) should be compared with the new form of the Weinberg equation for $j = 1$ spinors in a coordinate representation (Ahluwalia *et al.*, 1993; Ahluwalia and Goldman, 1993).

One can incorporate ρ spinor states in equations by using the identities (48a,b) of Ahluwalia (1994b):

$$\rho^S_\uparrow(p^\mu) = -i\lambda^A_\downarrow(p^\mu), \qquad \rho^S_\downarrow(p^\mu) = +i\lambda^A_\uparrow(p^\mu) \tag{63}$$

$$\rho^A_\uparrow(p^\mu) = +i\lambda^S_\downarrow(p^\mu), \qquad \rho^A_\downarrow(p^\mu) = -i\lambda^S_\uparrow(p^\mu) \tag{64}$$

Thus, one arrives at

$$\hat{p}\lambda^S_{\uparrow\downarrow}(p^\mu) + m\rho^A_{\uparrow\downarrow}(p^\mu) = 0, \qquad \hat{p}\lambda^A_{\uparrow\downarrow}(p^\mu) + m\rho^S_{\uparrow\downarrow}(p^\mu) = 0 \tag{65}$$

$$\hat{p}\rho^S_{\uparrow\downarrow}(p^\mu) + m\lambda^A_{\uparrow\downarrow}(p^\mu) = 0, \qquad \hat{p}\rho^A_{\uparrow\downarrow}(p^\mu) + m\lambda^S_{\uparrow\downarrow}(p^\mu) = 0 \tag{66}$$

It is also useful to note the connection of type II spinors $\lambda(p^\mu)$ and $\rho(p^\mu)$ with the type I Dirac bispinor $\psi^D(p^\mu)$ and its charge conjugate $(\psi^D(p^\mu))^c$:

$$\lambda^S(p^\mu) = \frac{1-\gamma_5}{2}\psi^D(p^\mu) + \frac{1+\gamma_5}{2}(\psi^D(p^\mu))^c \quad (67)$$

$$\lambda^A(p^\mu) = \frac{1-\gamma_5}{2}\psi^D(p^\mu) - \frac{1+\gamma_5}{2}(\psi^D(p^\mu))^c \quad (68)$$

$$\rho^S(p^\mu) = \frac{1+\gamma_5}{2}\psi^D(p^\mu) + \frac{1-\gamma_5}{2}(\psi^D(p^\mu))^c \quad (69)$$

$$\rho^A(p^\mu) = \frac{1+\gamma_5}{2}\psi^D(p^\mu) - \frac{1-\gamma_5}{2}(\psi^D(p^\mu))^c \quad (70)$$

Equations (65), (66) can then be rewritten in a form with type I spinors:

$$(\hat{p}+m)\psi^D_{\pm 1/2}(p^\mu) + (\hat{p}+m)\gamma_5(\psi^D_{\pm 1/2}(p^\mu))^c = 0 \quad (71)$$

$$(\hat{p}-m)\gamma_5\psi^D_{\pm 1/2}(p^\mu) - (\hat{p}-m)(\psi^D_{\pm 1/2}(p^\mu))^c = 0 \quad (72)$$

$$(\hat{p}+m)\psi^D_{\pm 1/2}(p^\mu) - (\hat{p}+m)\gamma_5(\psi^D_{\pm 1/2}(p^\mu))^c = 0 \quad (73)$$

$$(\hat{p}-m)\gamma_5\psi^D_{\pm 1/2}(p^\mu) + (\hat{p}-m)(\psi^D_{\pm 1/2}(p^\mu))^c = 0 \quad (74)$$

So, we can consider the $(\psi^D_h(p^\mu))^c$ [or $\gamma_5\psi^D_h(p^\mu)$, or their sum] as the positive-energy solutions of the Dirac equation and $\psi^D_h(p^\mu)$ [or $\gamma_5(\psi^D_h(p^\mu))^c$, or their sum] as the negative-energy solutions. The field operator can be defined by

$$\Psi = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2p_0}\sum_h [(\psi^D_h(p^\mu))^c a_h \exp(-ip\cdot x) + \psi^D_h(p^\mu) b^\dagger_h \exp(ip\cdot x)] \quad (75)$$

A similar formulation has been developed by Nigam and Foldy (1956).

Let us note an interesting feature. We can obtain another interpretation [namely, $\psi^D(p^\mu)$ corresponds to the positive-energy solutions and $(\psi^D(p^\mu))^c$ to the negative ones] if we choose other overall phase factors in the definitions of the rest-spinors $\phi_L(\mathring{p}^\mu)$ and $\phi_R(\mathring{p}^\mu)$ [formulas (22) of Ahluwalia (1994b)]. The signs of the mass term depend on the form of the generalized Ryder–Burgard relation; if $\theta_1 + \theta_2 = \pi$, the signs are opposite. One can obtain the generalized equations (57)–(60) for an arbitrary choice of the phase factor. For $\lambda^S(p^\mu)$ spinors they are

$$\begin{cases} i\hat{p}\lambda^S_\uparrow(p^\mu) - m\mathcal{T}\lambda^S_\downarrow(p^\mu) = 0 \\ i\hat{p}\lambda^S_\downarrow(p^\mu) + m\mathcal{T}\lambda^S_\uparrow(p^\mu) = 0 \end{cases} \quad (76)$$

where

$$\mathcal{T} = \begin{pmatrix} e^{i(\theta_1+\theta_2)} & 0 \\ 0 & e^{-i(\theta_1+\theta_2)} \end{pmatrix} \tag{77}$$

and $m \neq 0$. In the case $\theta_1 + \theta_2 = \pm\pi/2$ we also have the correct physical dispersion, $p_0^2 - \mathbf{p}^2 = m^2$, for $\lambda(p^\mu)$ spinors.

Next, one can see from (61), (62) that neither $\lambda^{S,A}(x)$ nor $\rho^{S,A}(x)$ is an eigenfunction of the Hamiltonian operator (we have different chiral helicities in the "Dirac" equations). They are not in mass eigenstates. However, ψ^{D} and $(\psi^{\mathrm{D}})^c$ are in mass and helicity eigenstates. Nigam and Foldy (1956) showed that even without a resort to a plane-wave expansion, if the eigenvector $|\phi\rangle$ has the eigenvalue "-1" of the normalized Hamiltonian $\hat{H}/|E|$ in Hilbert space, then $|\phi^c\rangle$ has the eigenvalue "$+1$." This analysis is in accordance with the Feynman–Stückelberg interpretation of an "antiparticle" as a particle moving backward in time (Stückelberg, 1941; Feynman, 1949), which seems to be deeper than Dirac's hole concept, because the former permits us to describe bosons on an equal footing with fermions (Ahluwalia *et al.*, 1993; Ahluwalia and Goldman, 1993). Thus, one can come to the conclusion that matrix elements, e.g., $\langle\lambda^A_{-\eta}(0), |\lambda^S_\eta(t)\rangle$, have nonzero value at the time t [cf. equations (27), (28)]:

$$\langle\lambda^A_{-\eta}|\lambda^S_\eta(t)\rangle \sim \sin^2\left(\frac{Et}{\hbar}\right), \qquad \langle\lambda^S_{-\eta}|\lambda^S_\eta(t)\rangle \sim \cos^2\left(\frac{Et}{\hbar}\right) \tag{78}$$

$$\langle\lambda^S_{-\eta}|\lambda^A_\eta(t)\rangle \sim \sin^2\left(\frac{Et}{\hbar}\right), \qquad \langle\lambda^A_{-\eta}|\lambda^A_\eta(t)\rangle \sim \cos^2\left(\frac{Et}{\hbar}\right) \tag{79}$$

We are ready to ask whether the high-energy neutrino described by the field [equation (47) of Ahluwalia (1994b)]

$$\nu^{\mathrm{ML}} \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2p_0}\sum_\eta [\lambda^S_\eta(p^\mu)a_\eta(p^\mu)\exp(-ip\cdot x) + \lambda^A_\eta(p^\mu)a^\dagger_\eta(p^\mu)\exp(ip\cdot x)] \tag{80}$$

can oscillate from the state of one chiral helicity to another chiral helicity with the oscillation length of the order of the de Broglie wavelength, $\lambda = h/p$.

For the spin-1 case the situation differs in some aspects. Direct calculations yield a nondynamical quadratic (in projections of the linear momentum) equation:

$$\left[\zeta_\lambda \frac{\gamma_{11}p_2^2 + \gamma_{22}p_1^2 - 2\gamma_{12}p_1p_2}{\mathbf{p}^2 - p_3^2} + \mathbb{1}\right]\lambda(p^\mu) = 0 \tag{81}$$

It can also be written in the form[23]

$$\begin{pmatrix} -\mathbb{1} & \zeta_\lambda D^{(1,0)}\left(i\dfrac{[\sigma \times \mathbf{p}]_3}{\sqrt{\mathbf{p}^2 - p_3^2}}\right) \\ \zeta_\lambda \Theta_{[1]} D^{(0,1)}\left(i\dfrac{[\sigma \times \mathbf{p}]_3}{\sqrt{\mathbf{p}^2 - p_3^2}}\right)\Theta_{[1]} & -\mathbb{1} \end{pmatrix} \lambda(p^\mu) = 0 \quad (82)$$

which is obtained by using, e.g., the technique of Novozhilov (1975), $D^{(J,0)}(A)$ are the Wigner functions for the $(J, 0)$ representation, $D^{(0,J)}(A)$, for the $(0, J)$ representation.

If we take another formulation of the Burgard–Ryder relation (45), we have[24]

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^S_\uparrow(p^\mu) - m^2\lambda^S_\downarrow(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \rho^S_\uparrow(p^\mu) - m^2\rho^S_\downarrow(p^\mu) = 0 \quad (83)$$

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^S_\downarrow(p^\mu) - m^2\lambda^S_\uparrow(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \rho^S_\downarrow(p^\mu) - m^2\rho^S_\uparrow(p^\mu) = 0 \quad (84)$$

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^S_\rightarrow(p^\mu) + m^2\lambda^S_\rightarrow(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \rho^S_\rightarrow(p^\mu) + m^2\rho^S_\rightarrow(p^\mu) = 0 \quad (85)$$

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^A_\uparrow(p^\mu) + m^2\lambda^A_\downarrow(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \rho^A_\uparrow(p^\mu) + m^2\rho^A_\downarrow(p^\mu) = 0 \quad (86)$$

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^A_\downarrow(p^\mu) + m^2\lambda^A_\uparrow(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \rho^A_\downarrow(p^\mu) + m^2\rho^A_\uparrow(p^\mu) = 0 \quad (87)$$

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^A_\rightarrow(p^\mu) - m^2\lambda^A_\rightarrow(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \rho^A_\rightarrow(p^\mu) - m^2\rho^A_\rightarrow(p^\mu) = 0 \quad (88)$$

There exist identities analogous to (63), (64). For instance, with the choice of the phase factors, corresponding to that in (83)–(88) we have[25]

$$\rho^S_\uparrow(p^\mu) = +\lambda^S_\downarrow(p^\mu), \quad \rho^S_\downarrow(p^\mu) = +\lambda^S_\uparrow(p^\mu), \quad \rho^S_\rightarrow(p^\mu) = -\lambda^S_\rightarrow(p^\mu) \quad (89)$$

$$\rho^A_\uparrow(p^\mu) = -\lambda^A_\downarrow(p^\mu), \quad \rho^A_\downarrow(p^\mu) = -\lambda^S_\uparrow(p^\mu), \quad \rho^A_\rightarrow(p^\mu) = +\lambda^A_\rightarrow(p^\mu) \quad (90)$$

Therefore,

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^S_{\uparrow\downarrow\rightarrow}(p^\mu) - m^2\rho^S_{\uparrow\downarrow\rightarrow}(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \lambda^A_{\uparrow\downarrow\rightarrow}(p^\mu) - m^2\rho^A_{\uparrow\downarrow\rightarrow}(p^\mu) = 0 \quad (91)$$

$$\gamma_{\mu\nu}p^\mu p^\nu \rho^S_{\uparrow\downarrow\rightarrow}(p^\mu) - m^2\lambda^S_{\uparrow\downarrow\rightarrow}(p^\mu) = 0, \quad \gamma_{\mu\nu}p^\mu p^\nu \rho^A_{\uparrow\downarrow\rightarrow}(p^\mu) - m^2\lambda^A_{\uparrow\downarrow\rightarrow}(p^\mu) = 0 \quad (92)$$

Applying relations between type II and type I spinors similar to (67)–(70) except for $\rho^S \leftrightarrow \rho^A$, we obtain

[23] We use the notation in terms of the Barut–Muzinich–Williams matrices here (Barut *et al.*, 1963).

[24] Again, one can obtain the opposite signs in the equations if one takes $\delta_1 + \delta_3 = \pi$ for $\phi_L(\mathring{p}^\mu)$ and correspondingly for $\phi_R(\mathring{p}^\mu)$.

[25] Compare with formulas (21a)–(21c) in Ahluwalia *et al.* (1994a) and (48), (49) in Dvoeglazov (1994f). Thus, the form of these relations depends on the choice of the spinorial basis, normalization and is governed by the covariance of the theory under discrete symmetries.

$$(\gamma_{\mu\nu}p^\mu p^\nu - m^2)\psi^D(p^\mu) + (\gamma_{\mu\nu}p^\mu p^\nu - m^2)\gamma_5(\psi^D(p^\mu))^c = 0 \qquad (93)$$

$$(\gamma_{\mu\nu}p^\mu p^\nu + m^2)\gamma_5\psi^D(p^\mu) - (\gamma_{\mu\nu}p^\mu p^\nu + m^2)(\psi^D(p^\mu))^c = 0 \qquad (94)$$

$$(\gamma_{\mu\nu}p^\mu p^\nu - m^2)\psi^D(p^\mu) - (\gamma_{\mu\nu}p^\mu p^\nu - m^2)\gamma_5(\psi^D(p^\mu))^c = 0 \qquad (95)$$

$$(\gamma_{\mu\nu}p^\mu p^\nu + m^2)\gamma_5\psi^D(p^\mu) + (\gamma_{\mu\nu}p^\mu p^\nu + m^2)(\psi^D(p^\mu))^c = 0 \qquad (96)$$

This tells us that ψ^D [or $\gamma_5(\psi^D)^c$] should be considered as the positive-energy solutions of the modified Weinberg equation (Ahluwalia *et al.*, 1993; Ahluwalia and Goldman, 1993) and $(\psi^D)^c$ (or $\gamma_5\psi^D$) as the negative-energy ones. The analogs of equations (61), (62) can be written as

$$\gamma^{\mu\nu}\partial_\mu\partial_\nu\lambda_\eta(x) + \wp_{S,A}m^2\lambda_{-\eta}(x) = 0 \qquad (97)$$

$$\gamma^{\mu\nu}\partial_\mu\partial_\nu\rho_\eta(x) + \wp_{S,A}m^2\rho_{-\eta}(x) = 0 \qquad (98)$$

where $\wp_{S,A} = \pm 1$; the plus sign is for positive-energy solutions $\lambda^S(p^\mu)$ [or $\rho^S(p^\mu)$] and the minus sign is for negative-energy solutions $\lambda^A(p^\mu)$ [or $\rho^A(p^\mu)$]. This refers to $\eta = \uparrow$ or $\eta = \downarrow$. For $\eta = \rightarrow$, it is easy to see that equations (85), (88) have the opposite signs for the mass terms.

The presence of $\wp_{\uparrow\downarrow}$ in the $j = 1/2$ case or $\wp_{S,A}$ in the $j = 1$ case suggests that we have obtained examples of FNBWW-type quantum field theory. The analysis of the field operators in the Fock space reveals that the fermion and its antifermion can possess the same intrinsic parities (Nigam and Foldy, 1956; Ahluwalia, 1994d). Bosons described by (93)–(96) are found (Ahluwalia *et al.*, 1993; Ahluwalia and Goldman, 1993) to be able to carry opposite intrinsic parities, depending on the choice of the field operator.

4. CONCLUDING REMARKS

In this paper I have presented an overview of the theory of truly neutral particles. The question of applicability of the new constructs in the $(j, 0) \oplus (0, j)$ representation space to neutrino physics has been discussed. The connection of the new models with the theories envisaged by Foldy and Nigam (1956) and Bargmann, Wightman, and Wigner (see Wigner, 1962) has been found. The particle properties with respect to the operation of parity discussed in the present paper [and in Ahluwalia *et al.* (1993, 1994a,b), Ahluwalia and Goldman (1993), and Ahluwalia (1994a,b)] are unusual. In fact, it was shown that these properties depend on the choice of the field operator. Moreover, it was found that the physical content depends on the choice of the spinorial basis. Research in the framework of other constructs (Weinberg, 1964a,b, 1969; Tucker and Hammer, 1971; Dvoeglazov, 1994b–e) in the $(1, 0) \oplus (0, 1)$ representation space deserves further elaboration.

Unfortunately, the present experimental data do not yet permit us to make reliable conclusions on the sufficiency of the Standard Model (and its limits). However, the wide interest in neutrino physics in the theoretical community and the forthcoming experimental facilities—SUPER-KAMIOKANDE, SNO (Sudbury), BOREXINO, ICARUS (CERN-Gran Sasso), HELLAZ, HERON [see, e.g., the proceedings of the recent neutrino conference (Anon, 1994)]—leave us with hope that the puzzles of mysterious neutral particles can be resolved in a short time.

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